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Mathematical Models in Tasks of Construction

Alexander N. Koshev[a], Valentina V. Kuzina[b,]*

[a]*Penza State University of Architecture and Construction, 28 German Titov St., Penza, 440028, Russia*

Abstract

To solve construction problems methods of mathematical modeling are used. The examples of various tasks solving are given. Calculations of optimum parameters of building structures, passive impurities transfer in water and air and others are presented. The information system for predicting the properties of building materials protection against radiation are described. The problems were solved by the authors using mathematical models.

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Keywords: mathematical simulation; optimization; prediction; construction tasks;

1. Introduction

The problems solving in construction by means of modeling are presented in the article. Methods of mathematical simulation made with the assistance of authors are shown. They are optimization of the amount of constructions of port structures; calculation of transfer of impurity in different environments and concentration of metals in sewage and prediction of properties of construction materials. Research objective to show the use of mathematical modeling with the aim of various construction problems solving.

* Corresponding author. Tel.: +79093165597, fax: (8412) 49-72-77;
E-mail address: kuzina@pguas.ru

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2. Main part of the article

2.1. Mathematical modeling and calculation of structures of an optimal size for hydraulic structures in Porto

Structures of massive masonry with different types of add-ons which are the most common port facilities for the Northern regions of Russia and other countries possess one essential drawback high cost, which is a direct consequence of their high material consumption [1]. The task of calculation and optimization of this design can only be solved using mathematical modelling and optimization techniques. This problem showed be solved by method of optimization adopted for the calculation of port hydraulic structures. The optimum scheme was chosen from the condition of minimum cost designs for given building conditions. Independent parameters: the width of an underwater part of X_1 and the width of a superstructure of X_2 are selected (see Fig. 1). A set of calculations and their composition are fully consistent with current regulations. At the same time, the following inspections were carried out: 1) stability of a superstructure on the base – according to the diagram of plane shift; 2) restrictions of values of the squeezing and pulling stresses – on contact with superstructure and base; 3) stability of a construction according to the diagram of plane shift – on contact with construction and bed; 4) restrictions of value of the squeezing and pulling stresses on contact with construction and base.

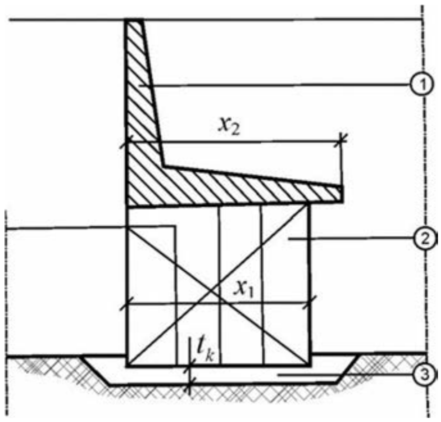

Fig. 1. Estimated diagram: 1 – superstructure (monolithic or precast concrete); 2 – underwater part (massive laying); 3 – base (stone bed).

Formalizing a physical problem definition, it is possible to write the task of optimization in the following way: to minimize construction cost $S(X_1, X_2)\rightarrow$min in case of restrictions $f_i(X_1,X_2)\leq 0$, $i=1,\ldots,m$. The total cost consists of the cost of each of construction elements: costs of the base, cost of a superstructure and cost of backfilling. Type of the functions $f_1(X_1, X_2),\ldots,f_m(X_1, X_2)$ is extremely bulky and for this reason can't be given in an analytical way. The functions $S(X_1, X_2)$, $f_m(X_1, X_2)$ aren't linear concerning coordinates of x_1, x_2 and, therefore, the solvable task is the task of non-linear programming. To solve this problem we used the method of penalty functions that is method of an external point with the function for the unconditional minimization of the following formulae: $F(X_1,X_2) = S(X_1,X_2) + t_k\Sigma(|f_i(X_1,X_2)| - f_i(X_1,X_2))/2$. To solve the problem, the function $F(X_1,X_2,t_k)$ must be minimized on an unlimited, strictly increasing sequence of numerals $\{t_k\}$. Specific functions $f_i(X_1,X_2)$, $S(X_1,X_2)$ are established and good results can be obtained by minimizing $F(X_1,X_2)$ and by the method of Hook and Jeeves that is modified by the method of "the descent along each coordinate". Since the function $S(X_1,X_2)$ is strictly increasing with respect to the variables, initial values, it is advisable to take the minimum possible value for this structure. At the same time, as a rule, initial approach doesn't get to the area of performance of restrictions – it has defined the choice of the solution method of a task. Performance of all restrictions can be considered as the conditional end of process of minimization of the function $F(X_1,X_2)$. On the basis of stated above the program complex for the computer is made. Development and deployment at design institute of the computer program for the solution of this task yielded the following results: labor productivity of a designer increased by 35 times, the cost of a construction decreased by 10%.

2.2. Model of mathematical calculation of passive impurity transfer in limited space

One of actual problems of heating and ventilating equipment is calculation of fields of temperatures and concentration of impurity in streams of air or water in the limited room [2].

For many practical cases the task can be reduced to calculation of passive Φ size transfer (temperature, concentration, enthalpy) in a turbulent stream. We will consider a case when the source occupies only part of a surface of border, i.e. it is significantly less than the sizes of main current. We suggest dividing the solution of a task into two stages by drawing up mathematical model. At the first stage by the known method the fields of average speeds and characteristics of turbulence are defined. For many important engineering cases (interfaces, wall streams, streams in pipes, channels, etc.) the specified sizes are known, and then calculations for the first stage significantly become simpler. At the second stage fields for function Φ on the basis of the solution of the equation of transfer of elliptic type are calculated: $\partial(\rho u\Phi)/\partial x + 1/r\cdot\partial(r\rho v\Phi)/\partial r = \partial(\Gamma_\Phi\cdot\partial\Phi/\partial r)/\partial x + 1/r\ \partial(r\Gamma_\Phi\cdot\partial\Phi/\partial r)/\partial r$, where x and r – cylindrical coordinates; ρ – density; Γ_Φ – turbulent transfer coefficient; u and v – projections of average speeds in the directions x and r. This equation was solved by the finite difference method by approximating the first and second derivatives of the coefficients and variables of their discrete counterparts, with the use of regular square grid. Thus, the problem is reduced to solving a system of linear algebraic equations of high dimensionality. To solve the resulting system of linear algebraic equations used Seidel method and conjugate gradient. Seidel method provided a better convergence for this class of problems. Special computing procedures written to implement the method. The calculation results of fields function Φ, the physical meaning of which can be the temperature, enthalpy, impurity concentration, showed good agreement between model results and experimental data.

2.3. The information system for predicting the properties of building materials protection against radiation

The development of nuclear technology, radioactive waste disposal in the regions, and other man-made causes lead to the need to develop and implement new efficient building materials for protection from powerful sources of ionizing radiation [3]. An important step in solving the problem is to develop new and choose existing of materials with low cost and with the best barrier properties to reduce the γ-radiation for protection of the population. It has been established that the interaction between macroscopic cross-section or a linear coefficient of attenuation depends on the density and the chemical elements in the building materials. Therefore, one of the main problems is the problem of finding regularities that govern specific to this case quantitative relationships and ties. To select known building materials and design new ones, based on the existing data on the dependence of material properties and attenuation coefficient, and for predicting the properties of new materials information system, which provides a database for studying the properties of protective materials have been developed, as well as mathematical models that predict the composition of materials with the right defensive abilities. Major barrier properties of building materials (linear attenuation coefficient γ-radiation mass attenuation coefficient of the relaxation length, etc.), the characteristic material properties (thermal conductivity, tensile strength, porosity, etc.) and background information on chemical elements ace placed in to the database. In order to solve the problem of constructing new building materials regression models showing the influence of the composition of materials on the performance of protective properties have been developed. The database is used containing protective properties of construction materials depending on concentration of chemical elements. Regression analysis allows us to investigate the influence of factors on the target parameters. This in turn makes it possible to predict the change of the target parameters depending on changes in factors. For example, increasing the amount of oxygen in ordinary concrete with boron 20% increases the linear attenuation coefficient of gamma radiation by 12%, which in turn improves the protective properties of the concrete. It was also found that the greater the mass of the chemical element and the more of it is contained in the material, the higher the radiation protection is.

2.4. Mathematical modelling of the system CaO-SiO_2-H_2O

When designing concrete mixes is important to investigate the processes occurring in the system CaO-SiO_2-H_2O, which occur during setting and hardening silicate binders [4]. Mathematical modelling shows that in such a system damped and undamped oscillations intermediate unstable compounds concentrations appear, as well system of differential equations for modelling process is unstable according to Lyapunov. It was also found that the kinetics of CaO and SiO_2 concentrations in the liquid phase of the system is similar to microkinetic intermediate unstable compounds: X_1, X_2 and X_3. X_1 – H is radicals and OH; X_2 – short-centres on the surface of the silicon atoms SiO_2 particles; X_3 – short-centres on the oxygen atoms of the surface of SiO_2 particles. Influence of the time interval on the amplitude of the oscillation and the oscillation frequency of the intermediate concentrations of volatile compounds studied by mathematical modelling. The study was based on a system of linear differential equations relating to the concentration and constants of speeds of transformation components of chemical reactions in the system:

$$\begin{cases} dX_1/dt = k_1X_1 + (k_4 - k_2)X_2 + (k_5 - k_3)X_3; \\ dX_2/dt = k_2X_1 - k_4X_2 - k_6X_2X_3; \\ dX_3/dt = k_3X_1 - k_5X_3 - k_6X_2X_3. \end{cases}$$

The system is defined on time slot from the tenth fractions of a second to one and more than a second. By calculations it is set that value of time slot influences the frequency and the form of oscillations of concentration X_1, X_2 and X_3 during synthesis of hydrosilicates of calcium. Data retrieved allow to claim that elementary stages of processes of education and interaction of the active particles of X_1, X_2, X_3 in time proceed non-linearly, and constants of speeds during synthesis in certain limits change. The system is defined on time slot from the tenth fractions of a second to one second and more. By calculations it is set that value of time slot influences the frequency and the form of oscillations of concentration X_1, X_2 and X_3 during synthesis of hydrosilicates of calcium. Findings allow to claim that elementary stages of processes of education and interaction of the active particles of X_1, X_2, X_3 in time proceed non-linearly, and constants of speeds during synthesis in certain limits change.

2.5. Mathematical modelling of moisture distribution in wet paint-ties of materials

Moist material is considered as a porous medium with allocation divided by the volume of the effective characteristics, such as porosity, heat conductivity, adsorptive properties, and others [5]. In the process of hydration of the material environment in each point of the porous space, the reaction occurs as adsorption of moisture, which has its macro and microscopic patterns. Changes in moisture and validating a shared environment in time obeys the law of conservation of matter, after detailing and transformations we obtain the following differential equation in partial derivatives: $\partial(\varphi)/\partial t = -\mathrm{div}(\varphi\cdot\boldsymbol{w}) - D\cdot(d^2\varphi/dx^2 + d^2\varphi/dy^2 + d^2\varphi/dz^2) + kF_{nc}f(\varphi)$, where k – the constant adsorption rate per unit area; F_{nc} – specific surface area per unit volume of the porous medium, m^2/m^3. The results of physical and mathematical modelling made it possible to record the dependence (curve moisture adsorption) $W_p = f(\varphi)$ as analytically specified function, which is characterized by a set of parameters, and their numerical values that are determined by a specific mechanism of moisture adsorption process:

$$W_p = k_2 \cdot \ln(1+ \varphi)/\ln(k_3 + \varphi) \cdot \exp(k_1(\varphi - \varphi_c)) + W_n / ((1+W_n / W_m -1) \cdot \exp(- k\varphi)),$$

where k_1, k_2, k_3 and φ_c – some constants whose values are found by us with the use of mathematical methods of experimental data processing. Estimates suggest that the mathematical relationship accurately reflects the equilibrium process of interaction, for example, textile fibers with air-conditioning with a maximum deviation of about 5% of the experimental curve.

2.6. The monitoring system of possible spread of man-made pollution water in the region on the basis of mathematical models

Three main options of a mass transfer of the contaminating substance were considered: diffusion; convective and diffusion; preferentially convective. For these purposes one-dimensional, two-dimensional and three-dimensional mathematical models are constructed [6]. In the conditions of one-dimensional diffusion when the mass transfer occurs in one prevailing direction, two options are considered: 1) diffusion from a dot source with known limited amount of substance; 2) diffusion from a constant source with a known speed of the expiration of substance. The mathematical description of process contamination in both cases is the diffusion equation in a liquid medium, the second reflecting Fick's law with the appropriate initial and boundary conditions. Calculation formulas for determining pollutant concentrations obtained using the Poisson integral. A two-dimensional diffusion problem arises, for example, when the contaminant enters the pond through some portion of the reservoir boundary, sufficiently small compared to the size of the entire border.

In this case, two-dimensional diffusion equation was used for modelling using specific initial and boundary conditions, and the solution is obtained by Fourier transformation with the nucleus of a special kind. The case of three-dimensional spherical diffusion is considered on the example of the solution of an actual task when the source of pollution is at the beginning of coordinates (in the depth of a reservoir) and produces in unit of time a substance quantity q=const. In this case, the diffusion equation is written out in spherical coordinates. Standard initial and boundary conditions are supplemented by the boundary conditions derived by us in the vicinity of the source of pollution. We made an attempt to systematize the potential environmental situation and build corresponding mathematical models that allowed us to obtain quantitative algorithms for solving the tasks of monitoring and develop the information system for calculating the concentrations of pollutants in the water system in the region.

3. Conclusions

The reviewed examples show a possibility of the solution of actual problems of construction by methods of mathematical simulation. The scientific research shows that some problems of construction are solved only by modelling as they exclude all types of experiment besides a computer one. Some problems solving needs modelling as the only convenient and economic way of solving.

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